ference of the absorption coefficients, leading to the Borrmann effect, will be smaller and so the anomalous transmission will be less for this orientation in agreement with observations in zone axes Kikuchi-pattern shown by UYEDA and NONOYAMA [17] and recently by FUJIMOTO et. al. [18].

It is a great pleasure for me to thank Dr. KAMBE for many discussions and also Dr. WAGENFELD and Prof. FUJIMOTO for suggesting the positron calculations.

[17] R. UYEDA and M. NONOYAMA, Jap. J. Appl. Phys. 7, 200 [1968].

[18] F. FUJIMOTO, K. KOMAKI, and Y. UCHIDA, Phys. Stat. Sol. (a) 8, K71 [1971].

M1 Transitions in the Reaction ^{6}Li(γ,^{2}H)^{4}He

K. H. PASSLER

Institut für Theoretische Physik der Justus-Liebig-Universität Gießen, Germany

(Z. Naturforsch. **28a**, 8—11 [1973]; received 10 October 1972)

It is shown that the admixture of a D-state in the wave function for the relative motion of the α-particle and the deuteron in the nucleus ^{6}Li leads to M1 transitions. Their mixture with E2 transitions does not lead to an asymmetry of the differential cross section of the reaction ^{6}Li(γ,^{2}H)^{4}He about 90°, and they are negligibly small.

According to a selection rule [1] which follows from the charge symmetry of nuclear forces, E1 transitions in self-conjugate nuclei without change of the total isospin quantum number are forbidden. However it was shown [2] that in the derivation of this rule an assumption was made which is not valid in the cluster model. This assumption reads that the charge-to-mass ratios of all groups of nucleons in a nucleus are equal.

The effects of clustering are expected to be relatively strong in the nucleus ^{6}Li. Calculations show [3] that the binding energies of the ^{2}H and ^{4}He clusters are very close to the binding energies of the corresponding free particles. So it is a good approximation to use the masses of the free particles for the masses of these clusters.

A possible violation of this isospin selection rule can be tested in the reaction

$$^6\text{Li}(\gamma, {}^2\text{H})^4\text{He}. \qquad (1)$$

First we will assume that in the ground state of ^{6}Li the two clusters are in a relative S-state. Then there are no M1 transitions. Therefore, if the selection rule is valid only E2 transitions can contribute, so the differential cross section is predicted to be symmetrical about 90°. On the other hand, if clustering effects are important, the selection rule is violated and E1 transitions can occour. The mixture of these E1 transitions with the E2 transitions then would cause an asymmetry of the differential cross section.

However, the nonvanishing quadrupole moment of the nucleus ^{6}Li indicates that there may be an admixture of a D-state in the relative motion. This would allow M1 transitions, which then would lead to interference terms with the E2 transitions. Thus, even if the isospin selection rule is not violated, the angular distribution of the reaction products may not be that of a pure E2 transition.

Reprint requests to Dipl.-Phys. KARL-HEINZ PASSLER, Institut für Theoretische Physik, *D-6300 Gießen*, Leihgesterner Weg 108.

[1] E. K. WARBURTON and J. WENESER, in: Isospin in Nuclear Physics, edited by D. H. WILKINSON, North Holland Publishing Company, Amsterdam 1969, p. 215.

[2] V. I. MAMASAKHLISOV and T. S. MACHARADZE, Sov. J. Nucl. Phys. 9, 198 [1969].

[3] E. W. SCHMID, Y. C. TANG, and K. WILDERMUTH, Phys. Letters 7, 263 [1963].

[4] J. L. GAMMEL, B. J. HILL, and R. M. THALER, Phys. Rev. 119, 267 [1960].

To calculate the cross section of the reaction (1) we use the wave function

$$\psi^{\mathrm{i}}(\boldsymbol{R}) = a\psi^{\mathrm{i}}_{01}(R)\,[0,1]_{1,Mi} + b\psi^{\mathrm{i}}_{21}(R)\,[2,1]_{1,Mi} \qquad (2)$$

for the relative motion of both clusters in the initial state and the wave function

$$\psi^{\mathrm{f}}(\boldsymbol{R}) = \sum_{LMJ} 2\pi i^L\, Y_{LM}^{*}(q) < L\, M\, 1\, m \mid L\, 1\, J\, M+m > \psi^{\mathrm{f}}_{LJ}(R)\,[L,1]_{J,M+m} \qquad (3)$$

for the final state. Here ψ^{i}_{LJ} and ψ^{f}_{LJ} are the radial wave functions for the initial and final state which belong to the quantum numbers L and J of the system ^{2}H+^{4}He. The relative radius vector and wave number of the two clusters are denoted by $\boldsymbol{R}$ and $\boldsymbol{q}$. The symbol $[L,1]_{J,M}$ stands for

$$\sum_{m_1,m_2} < L\, m_1\, 1\, m_2 \mid L\, 1\, J\, M > Y_{Lm_1}(q)\,\chi_{1m_2}$$

where χ_{1m} is the spin function of the deuteron. The coefficients a and b of the S- and D-functions are interrelated by the equation $a^2 + b^2 = 1$.

The wave functions (2) and (3) are not antisymmetrisized. It is possible to compensate for this dificiency to a certain degree by choosing a suitable optical potential for the relative motion. The potential has to become strongly repulsive when the two clusters are in a relative S-state and penetrate each other very far. In Fig. 1 a potential is shown which is an improved version of a potential that was calculated from the relative wave function for ^{6}Li given by SCHMID et al. [3]. It is chosen so as to give the correct binding energy of ^{6}Li.

The Hamilton operator for the reaction (1) which allows for E1, E2, and M1 transitions is

$$H = -i\, e_0\, (4\pi\varepsilon_0)^{-\frac{1}{2}}\, (2\pi E_\gamma/V)^{\frac{1}{2}}\, (H_{\mathrm{E}1} + H_{\mathrm{E}2} + H_{\mathrm{M}1})$$

where

$$H_{\mathrm{E}1} = \left((M_\alpha - 2M_{\mathrm{d}}) / (M_\alpha + M_{\mathrm{d}}) \right) (\boldsymbol{uR}), \qquad (4)$$

$$H_{\mathrm{E}2} = (i/3)\, (\boldsymbol{uR})\, (\boldsymbol{kR}), \quad \text{and}$$

$$H_{\mathrm{M}1} = (\hbar/2E_\gamma M_{\mathrm{p}} i)\, (\mu_{\mathrm{p}} + \mu_{\mathrm{n}} - \tfrac{1}{2})\, (\boldsymbol{k} \times \boldsymbol{u})\, \boldsymbol{S}\,.$$

The masses of ^{4}He, ^{2}H, and the proton are designated by M_α, M_{d}, and M_{p}, whilst $\boldsymbol{k}$, $\boldsymbol{u}$, and E_γ are the wave number, polarization vector, and energy of the γ-quants. The magnetic moments μ_{p} and μ_{n} of the proton and neutron are measured in units of nuclear magnetic moments. The normalization volume of the γ-quants is denoted by V.

From first order perturbation theory we obtain the differential cross section for the process (1). The results of the E1 and E2 transitions, and of their mixture (all with the S-state as initial state) are given in [2]. Here they need only to be multiplied by the factor $a^2 \approx 1$. For the mixture of the M1 transitions with the D-state as initial state and the E2 transitions with the S-state as initial state we obtain

$$\mathrm{d}\sigma/\mathrm{d}\Omega = ab\,\sqrt{5}\,(18\hbar c_0^3 E_{\mathrm{b}} M_{\mathrm{p}})^{-1}\, e_0^2 (4\pi\varepsilon_0)^{-1}\, \beta^3\, \gamma^2\, (\gamma-1)^{\frac{1}{2}}\, (\mu_{\mathrm{n}} + \mu_{\mathrm{p}} - \tfrac{1}{2}) \sum_{l}\sum_{J_1}\sum_{J_2} (2J_1+1)\,(2J_2+1)\,(2l+1)^{\frac{1}{2}}\, P_l(q)$$
$$\mathrm{Re}\,(<2\, J_1 \mid R^0 \mid 2\, 1>^{*} <2\, J_2 \mid R^2 \mid 0\, 1>)\, <2\,0\,2\,0 \mid 2\,2\,l\,0>\, <l\,0\,2\,1 \mid l\,2\,1\,1> \begin{Bmatrix} 1 & J_1 & 2 \\ 1 & 1 & 1 \end{Bmatrix} \begin{Bmatrix} 2 & l & 2 \\ J_2 & 1 & J_1 \end{Bmatrix} \begin{Bmatrix} J_1 & J_2 & l \\ 2 & 1 & 1 \end{Bmatrix} \qquad (5)$$

where $E_{\mathrm{b}} = 1.472$ MeV means the relative binding energy of the α-particle and the deuteron in the nucleus ^{6}Li, $\beta = (8\, E_{\mathrm{b}} M_{\mathrm{D}}/3\,\hbar^2)^{\frac{1}{2}}$, and $\gamma = E_\gamma/E_{\mathrm{b}}$.
The radial matrix elements are written as

$$<L\, J \mid R^n \mid L'\, J'> = \int_0^\infty \psi^{\mathrm{f}}_{LJ}(R)\, \psi^{\mathrm{i}}_{L'J'}(R)\, R^{n+2}\, \mathrm{d}R\,.$$

The term (5) in the differential cross section does not lead to an asymmetry about 90° because it contains only Legendre polonominals of even orders. Therefore, if in experiments the cross section is observed to be asymmetric this can be explained only by the admixture of E1 transitions.

The total cross section of the M1 transitions in the reaction (1) is

$$\sigma = (\pi\hbar/M_{\mathrm{p}}^2 c_0^3)\, e_0^2\, (4\pi\varepsilon_0)^{-1}\, \beta^3\, \gamma\, (\gamma-1)^{\frac{1}{2}}\, (\mu_{\mathrm{n}} + \mu_{\mathrm{p}} - \tfrac{1}{2})^2 \sum_J (2J+1) \left(\begin{Bmatrix} 1 & 2 & 1 \\ 1 & 1 & J \end{Bmatrix} \right)^2 \left| <2\, J \mid R^0 \mid 2\, 1> \right|^2. \qquad (6)$$

The probability for the D-state in the relative motion wave function, which is given by b^2, is interrelated with the quadrupole moment Q of ^{6}Li. To calculate Q, we use for the nucleus ^{6}Li the wave function

$$\Psi = \Phi_1(1234)\, \{a\psi_{01}(R)\,[0,1]_{1,1} + b\psi_{21}(R)\,[2,1]_{1,1}\}\,. \qquad (7)$$

Here, however, the D-state in the deuteron cluster is taken into account, so

$$[L,1]_{1,1} = \sum_{m_1,m_2} <L m_1\, 1\, m_2 \mid L\, 1\, 1\, 1> Y_{Lm_1}(\hat{R})\, [\Phi_2(56)]_{1,m_2}, \qquad (8)$$

where

$$[\Phi_2(56)]_{1,m} = u(r)\, Y_{00}(r) \mid 1m> + w(r) \sum_{m_1', m_2'} <2\, m_1'\, 1\, m_2' \mid 2\, 1\, 1\, m> Y_{2m_1}(r) \mid 1\, m_2'> \qquad (9)$$

is the deuteron wave function.

In (7), Φ_1 (1234) is the wave function for the α-cluster; it is assumed that this cluster is undeformed.

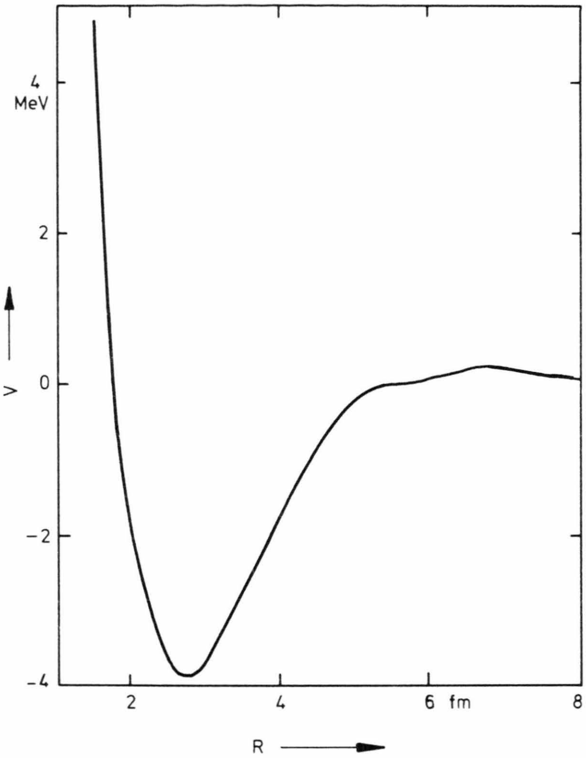

Fig. 1. An optical potential for the relative motion of the α and d clusters in the relative S-state is shown. The small barrier at 6.75 MeV is caused by the Coulomb potential. For small relative distances R the potential behaves like 131 MeV $\cdot$ fm$^2/R^2$.

A possible D-state admixture in this cluster would only lead to a contribution to Q in second order in b. In (9), $\boldsymbol{r}$ is the relative vector of the proton and the neutron in the deuteron.

With (7)–(9), we obtain for the quadrupole moment of ^{6}Li:

$$Q = a^2 Q_d + (8/15)\, 2^{-\frac{1}{2}}\, ab \int_0^\infty \psi_{01}(R)\, \psi_{21}(R)\, R^4\, dR + b^2 \left[(11/20)\, Q_d - (2/15) \int_0^\infty \psi_{21}^2(R)\, R^4\, dR \right]. \quad (10)$$

The quadrupole moment Q_d of the deuteron cluster is unknown. Calculations show [3] that the deuteron cluster in the nucleus ^{6}Li is considerably contracted. However, in these calculations only S-functions were used. In Fig. 2, as a function of the value of Q_d, the factor b is shown which gives the correct quadrupole moment Q of ^{6}Li, which is Q = –0.1 fm^2. The reasonable assumption $|Q_d| \lesssim 0.3$ fm^2 leads to $b^2 \lesssim 1.6 \cdot 10^{-4}$. A calculation of b by solving the Schroedinger equation for the relative motion gave $b = 0.024$. With that

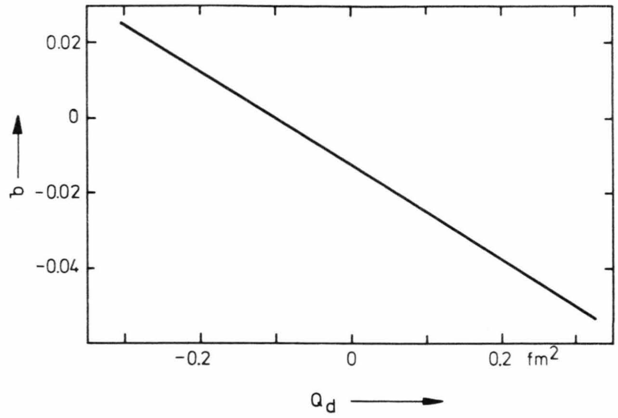

Fig. 2. As a function of the quadrupole moment Q_d of the deuteron cluster that coefficient b of the D wave function in ^{6}Li is shown, which gives the observed quadrupole moment of ^{6}Li.

for the S-state the potential shown in Fig. 1 was used, while the potential for the D-state, which was chosen to give the correct $\delta_{\frac{1}{2}}$ phase shifts of the ^{4}He(^{2}H, ^{2}H) ^{4}He scattering, was deeper by a factor of 1.2. The mixture of the states with different orbital angular momentum was caused by a tensor potential given in [4].

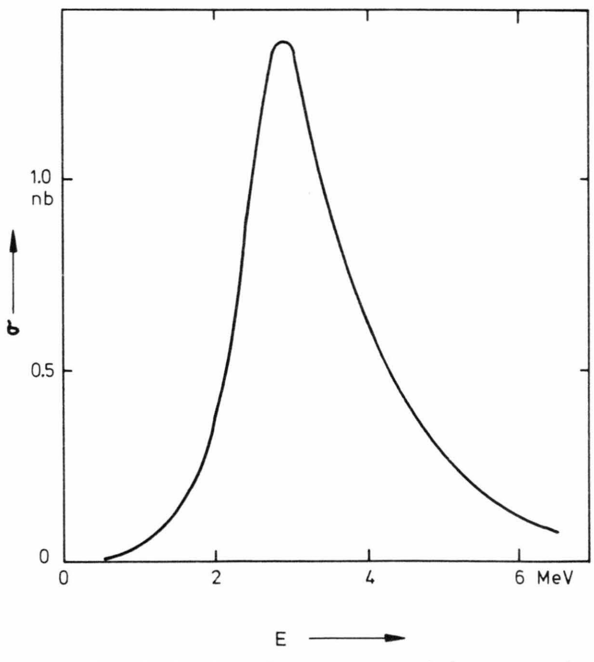

Fig. 3. The calculated total cross section of the M1 transitions in the reaction ^{6}Li(γ,^{2}H)^{4}He. The symbol E stands for the relative energy of ^{2}H and ^{4}He. The cross section is given in 10^{-9} barn.

In Fig. 3 the total cross section of the M1 transitions is shown which was obtained with the value $b = 0.024$. The potential used in these calculations was chosen to give the correct phase shifts of the $^4He(^2H, ^2H)^4He$ scattering between 2 and 10 MeV and to reproduce the resonance of the δ_3^2 shift at 1.07 MeV. This cross section is considerably smaller than that for E1 and E2 transitions [2] and thus it can be neglected. The reasons for its smallness are 1) the smallness of the D-state probability in the nucleus ^6Li and 2) an isospin selection rule [5] which diminishes the probability for M1 transitions with $\Delta T = 0$ in self-conjugate nuclei by a factor of about 100.

In the calculations of MAMASAKHLISOV and MACHARADZE [2] a square well potential with a well radius of 3.5 fm is used for the relative motiton. The depth is dependent on the angular momentum of the system $^4He + ^2H$; for the S-state it is 32.34 MeV. This potential is considerably different from the potential used in the present calculations and shown for the S-state in Fig. 1. The effect of the use of this long range potential with a repulsive hard core is mainly to increase those radial matrix elements which contain a power of the relative distance R. So the probability for E2 transitions is increased by a factor of about two in most parts of the energy region between 0 and 4 MeV relative energy of the reaction products. However, in the regions directly ahead and behind the first sharp maximum in the total E2 cross section at about 0.71 MeV, the cross section is diminished by a factor of up to five.

To summarize, we can conclude that the M1 transitions in the reaction (1) are negligible, and thus the asymmetry of the differential cross section is a reliable test for clustering effects in the nucleus ^6Li. On the other hand, the influence of the form of the optical potential used should not be underestimated, because the admixture of E1 transitions is expected to be most important just in the region ahead and behind the first peak in the cross section.

I wish to thank Professor Dr. G. EDER for his help and suggestions. The numerical calculations were performed at the Rechenzentrum der Justus Liebig-Universität, Giessen on a CDC 3300 computer.

[5] G. MORPURGO, Phys. Rev. **110**, 721 [1958].

Zum asymptotischen Verhalten der Lösungen der dreidimensionalen homogenen Wellengleichung

PAUL V. HOYNINGEN-HUENE

Sektion Physik der Universität München
Theoretische Physik

(Z. Naturforsch. **28a**, 11—13 [1973]; eingegangen am 23. Oktober 1972)

On the Asymptotic Behaviour of the Solutions of the Three-Dimensional Homogenous Wave Equation

In order to clarify the statement "A solution of the homogenous wave equation, that tends to zero at infinity faster than 1/r at all times, must be zero everywhere" three different limits are considered and investigated.

Bei HOYLE und NARLIKAR [1] findet man die Behauptung, daß eine Lösung der homogenen Wellengleichung, die zu allen Zeiten für $r \to \infty$ schneller als $1/r$ gegen null geht, identisch verschwindet. Die Richtigkeit dieser Behauptung ist für die Wheeler-Feynmansche Elektrodynamik wichtig [2]: Man möchte hier zei-

Sonderdruckanforderungen an Paul v. Hoyningen-Huene, Institut für Theoretische Physik der Universität Zürich, *CH — 8001 Zürich* (Schweiz).

[1] F. HOYLE u. J. V. NARLIKAR, Proc. Roy. Soc. London A 277, 1 [1964]. — J. V. NARLIKAR, Pure Appl. Chem. 22, 449 [1970].
[2] J. A. WHEELER, R. P. FEYNMAN, Rev. Mod. Phys. 17, 157 (1945); Rev. Mod. Phys. 21, 425 [1949].